The Electrogravitational Field of an Electrically Charged Mass Point and the Causality Principle in RTG

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In this paper, I determine the electrogravitational field produced by a charged mass point according to the Relativistic Theory of Gravitation. The Causality Principle in the Relativistic Theory of Gravitation will play a very important part in finding this field. The analytical form and the domain of definition, i.e the gravitational radius of the obtained solution, differ from that given by Einstein's General Relativity Theory.

Keywords: Relativistic Theory of Gravitation, Electrogravitational Fields, Causality Principle.

1 Introduction

The purpose of this paper is to find the electrogravitational field produced by a charged mass point in the framework of the Relativistic Theory of Gravitation (RTG).

In Section 2, I present the basics of this new relativistic theory of gravitation elaborated by Logunov and his co-workers (see Logonov & Mestvirishvili 1989 and Logunov 1997). In RTG, a gravitational field is determined unambiguously solving the complete system of RTG's Eqs. The Causality Principle (CP) in RTG, permits the selection of those solutions of RTG's Eqs. which can have physical meaning.

In Section 3, I present a formulation of the basic laws of electromagnetism in the vacuum, in the presence of the gravitational field. The presentation is based on the fact that the behaviour of an electromagnetic field can be formulated in a four dimensional manifold without supplementary mathematical structure (see Truesdell & Toupin 1960, Sections 266-275 and also Soós 1992 Sections 1-3).

The problem of finding the field of an electrically charged mass point was solved in the framework of Einstein's General Relativity Theory (GRT) by Nordström and Jeffrey (see Wang 1979, Section 56). At the beginning of Section 4, I present the solution obtained by them. But as was shown by Logunov and his co-workers, in GRT, one can not obtain an unique solution of the problem without introducing prior assumptions (see Logonov & Mestvirishvili 1989, Section12). Besides, the gravitational radius of the source point, as function depending on q^2 and m^2, where q and m representing the electric charge and the mass of the point source, has a discontinuity in $q^2 = m^2$.

In RTG, the considered problem was firstly analysed by Karabut & Chugreev (1987), but only assuming $m^2 \geq q^2$. In Section 5, I present this solution. I verify also that this solution satisfies CP in RTG, so, it's a solution which has physical sense.

Soós and me (2000) have reanalyzed the problem in RTG, considering also the possibility $q^2 > m^2$. It's important to analyse this case because for the electron just this variant is true. The analytical form of the solution found by us, as well as its domain of definition, i.e. the gravitational radius r_g, depend essentially on the relation existing between q^2 and m^2. But, I'll show in Section 6 that this solution doesn't fulfill CP in RTG. So, this solution can't be an acceptable solution in this theory.

I determine in Section 6, the unique solution of electrogravitational field produced by a charged mass point according to RTG. The obtained solution has the same analytical form for all order relations between q^2 and m^2. The gravitational radius depend on this relation but it's a continuous function depending on q^2 and m^2.

2 RTG's equations and the Causality Principle in RTG

RTG was constructed by Logunov and his co-workers (see Logonov & Mestvirishvili 1989 and Logunov 1997) as a field theory of the gravitational field within the framework of Special Relativity Theory (SRT). The Minkowski space-time is a fundamental space that incorporates all physical fields, including gravitation. The line element of this space is:

$$d\sigma^2 = \gamma_{mn}(x) dx^m dx^n, \tag{2.1}$$

where x^m, m =1, 2, 3, 4, is an admissible coordinate system in the underlying Minkowski space-time; $\gamma_{mn}(x)$ are the components of the Minkowskian metric in the assumed coordinate system.

The gravitational field is described by a second order symmetric tensor $\phi^{mn}(x)$, owing to the action of which un effective Riemannian space-time arises.

One of the basic assumption of RTG tells us that the behaviour of matter in the Minkowskian's space -time with metric $\gamma_{mn}(x)$, under the influence of the gravitational field $\phi^{mn}(x)$, is identical to its behaviour in the effective Riemannian space-time with metric $g_{mn}(x)$, determined according to the rules:

$$\tilde{g}^{mn} = \sqrt{-g} g^{mn} = \sqrt{-\gamma}\gamma^{mn} + \sqrt{-\gamma}\phi^{mn}, g = \det(g_{mn}), \gamma = \det(\gamma_{mn}). \tag{2.2}$$

Such interaction of the gravitational field with matter was termed the geometrization principle of RTG.

The behaviour of the gravitational field is governed by the following differential laws of RTG:

$$\begin{aligned} R^m_n - \frac{1}{2}\delta^m_n R + \frac{m_g^2}{2}\left(\delta^m_n + g^{mk}\gamma_{kn} - \frac{1}{2}\delta^m_n g^{kl}\gamma_{kl}\right) &= 8\pi T^m_n, & (2.3)\\ D_m \tilde{g}^{mn} &= 0, \quad m, n, p = 1, 2, 3, 4. & (2.4) \end{aligned}$$

Here R^m_n is Ricci's tensor corresponding to g_{mn} , $R = R^m_m$ is the scalar curvature, δ^m_n are Kronecker's symbols and T^m_n denotes the energy-momentum tensor of the sources of the gravitational field. In (2.4) D_m is the operator of covariant differentiation with respect to the metric γ_{mn}. Eqs. (2.3), (2.4) are covariant under arbitrary coordinate transformations with a nonzero Jacobian. All the field variables in the RTG depend on universal space-time coordinates of the

Minkowski space. The presence of mass terms in Eq. (2.3) allows unambiguously determining the space-time geometry and the density of the gravitational field energy-momentum tensor in the absence of matter. Eqs. (2.4) tell us that a gravitational field can have only the spin states 0 and 2. In Logonov & Mestvirishvili (1989), this represents one of the basic assumption of RTG. In Logunov (1997), these Eqs. which determine the polarization states of the field, are consequences of the fact that the source of the gravitational field is the universal conserved density of the energy-momentum tensor of the entire matter including the gravitational field. The graviton mass essentially affects the Universe's evolution and changes the character of the gravitational collapse.

In the present paper, because the graviton mass is negligibly small, we omitt the mass terms in Eq. (2.3). Hereafter, we use the relativistic system of units.

Eqs. (2.4) can be written in the following form (see Logonov & Mestvirishvili 1989, Appendix 1):

$$D_m \tilde{g}^{mn} = \tilde{g}^{mn}{}_{,m} + \gamma^n_{mp} \tilde{g}^{mp} = 0, \tag{2.4'}$$

where γ^n_{mp} are the components of the metric connection generated by γ_{mn} and the comma is the derivation relative to the involved coordinate.The causality principle (CP) in RTG is presented and analysed by Logunov (1997), Section 6.

According to CP any motion of a pointlike test body must have place within the causality light cone of Minkowski's space-time. According to Logunov's analysis CP will be satisfied if and only if for any isotropic Minkowskian vector u^m , i.e. for any vector u^m satisfying the condition:

$$\gamma_{mn} u^m u^n = 0, \tag{2.5}$$

the metric of the effective Riemannian space-time satisfies the restriction:

$$g_{mn} u^m u^n \leq 0 \tag{2.6}$$

According to CP of RTG only those solutions of the system (2.3), (2.4) can have physical meaning which satisfies the above restriction.

It's important to stress the fact that CP in the above form can be formulated only in RTG, because only in this theory, the space-time is Minkowskian and the gravitational field is described by a second order symmetric tensor field $\phi_{mn}(x)$, x^m being the admissible coordinates in the underlying Minkowskian space-time, x^1, x^2, x^3 being the space-like variables and x^4 being the time-like variable.

3 Electromagnetic field equations in RTG

The theory of electromagnetism is a very elegant and formally simple part of physics. The two principles of conservation set up as the basis for this theory, are the conservation of charge and the conservation of magnetic flux (see Truesdell & Toupin 1960, Sections 266-270, and Soós 1992, Sections 1-3). These two laws of conservation can be formulated in a four dimensional manifold independent of any geometry of space-time. The differential form of the so-called Maxwell-Bateman laws is (see Soós 1992, Sections 1-3):

$$\Psi^{mn}_{,m} = K^n \tag{3.1}$$

$$F^{mn}_{,m} = 0, \quad m, n, p = 1, 2, 3, 4, \tag{3.2}$$

where F^{mn} is a contravariant axial 2-vector density representing the electromagnetic field, Ψ^{mn} is a contravariant 2-vector density representing the electromagnetic induction and K^n is a contravariant 1-vector density representing the electric current and the charge density. All the involved fields in Eqs. (3.1), (3.2), depend on an admissible coordinate system (x^n) in the four dimensional manifold.

The conception of conservation as formulated here have a topological significance, being independent of the mathematics of length, time and angles. To interpret Eqs. (3.1), (3.2) in the familiar language of electromagnetic theory, a metric structure must be considered on the four dimensional manifold.

We assume now that our manifold is the Minkowski space-time of RTG. In this case we suppose that $(\alpha, \beta, \delta = 1, 2, 3)$:

$$F^{\alpha\beta} = \varepsilon^{\alpha\beta\delta} E_\delta, F^{\alpha 4} = B^\alpha, \Psi^{\alpha\beta} = \varepsilon^{\alpha\beta\delta} H_\delta, \Psi^{\alpha 4} = -D^\alpha, K^\alpha = j^\alpha, K^4 = \rho. \tag{3.3}$$

In the above Eqs. $\varepsilon^{\alpha\beta\delta}$ are the contravariant Ricci's permutation symbols, E_δ are the covariant components of the electric field and B^α the contravariant components of the magnetic field, corresponding to the selected Minkowskian coordinate system which may be inertial or not. Also, H_δ are the covariant components of the magnetic induction and D^α are the contravariant components of the electric induction, whereas j^α and ρ represent the contravariant components of the electric current and the density of the electric charge, respectively.

As a fundamental hypothesis of the relativistic electromagnetism in vacuum and excluding gravitation we assume that F^{mn} and Ψ^{mn} are connected by the Maxwell-Lorentz aether relation (see Truesdell & Toupin 1960, Section 280, and Soós 1992, Section 24):

$$\Psi^{mn} = \sqrt{-\gamma}\gamma^{mp}\gamma^{nq}\hat{F}_{pq}, \hat{F}_{pq} = \frac{1}{2}\varepsilon_{mnpq}F^{mn}, m, n, p, q = 1, 2, 3, 4, \tag{3.4}$$

where ε_{mnpq} are Ricci's covariant permutation symbols and $\hat{F}_{pq}$ is the dual of F^{mn}, being a skew-symmetric absolut tensor. From (3.4), in a Minkowskian inertial frame we get the familiar relations characterizing the vacuum:

$$D^\alpha = E_\alpha, \; H_\alpha = B^\alpha. \tag{3.5}$$

Let us assume now the presence of the gravitational field. Eqs. (3.1), (3.2) rest also true. Taking into account the geometrization principle of RTG , I assume that in the presence of the gravitational field the Maxwell-Lorentz aether relation takes the following form:

$$\Psi^{mn} = \sqrt{-g}g^{mp}g^{nq}\hat{F}_{pq}, \hat{F}_{pq} = \frac{1}{2}\varepsilon_{mnpq}F^{mn}, m, n, p, q = 1, 2, 3, 4, \tag{3.6}$$

Introducing (3.6) into (3.1), (3.2) and taking into account that for any skew-symmetric tensor X^{mn} we have the formula $\bigtriangledown_m X^{mn} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^m}\left(\sqrt{-g}X^{mn}\right)$, $\bigtriangledown_m$being the operator of covariant differentiation with respect to the metric g_{mn} , the laws describing the behaviour of an electromagnetic field in RTG can be written in the form:

$$\bigtriangledown_m \hat{F}^{mn} = J^n, \tag{3.7}$$

$$\bigtriangledown_i \hat{F}^{mn} + \bigtriangledown_m \hat{F}^{ni} + \bigtriangledown_n \hat{F}^{im} = 0, \tag{3.8}$$

where $J^n \equiv \frac{K^n}{\sqrt{-g}}$.

To get the energy-momentum tensor T_n^m of the electromagnetic field (in vacuum) in RTG we start with its expression in the relativistic electromagnetism excluding gravitation (see Moller 1972, Section 7):

$$\sqrt{-\gamma}T_n^m = -\frac{1}{4\pi}\hat{F}_{np}\Psi^{mp} + \frac{1}{16\pi}\hat{F}_{pq}\Psi^{pq}\delta_n^m. \tag{3.9}$$

Using again the geometrization principle of RTG and the assumed Maxwell-Lorentz aether relations (3.6), we can conclude that in RTG, T_n^m are given by:

$$T_n^m = -\frac{1}{4\pi}\hat{F}_{np}\hat{F}^{mp} + \frac{1}{16\pi}\hat{F}_{pq}\hat{F}^{pq}\delta_n^m,\ \hat{F}^{mp} = g^{mi}g^{pj}\hat{F}_{ij}. \tag{3.10}$$

In GTR, the electromagnetic field Eqs.(3.7), (3.8) and the energy-momentum tensor T_n^m of the electromagnetic field (3.10), have the same form as below, the important difference being that in RTG all fields depend on the coordinates in the underlying Minkowski space-time.

4 The electrogravitational field produced by a charged mass point in RTG

In the framework of GRT the problem of finding the field produced by a charged mass point with mass m and electric charge q, was solved by Nordström and by Jeffrey (see for example Wang 1979, Section 56).

The problem is static and spherically symmetric. The nonzero components of the Riemannian metric which represent the electrogravitational field, are the following:

$$g_{11} = -\frac{1}{1-\frac{2m}{r}+\frac{q^2}{r^2}},\ g_{22} = -r^2,\ g_{33} = -r^2\sin^2\theta,\ g_{44} = 1-\frac{2m}{r}+\frac{q^2}{r^2}. \tag{4.1}$$

In the above relations r and θ are two of the spherical coordinates $\{r,\theta,\varphi,t\}$ centered in the charged mass point, in which are written the components of the metric. The domains of definition for these coordinates are: $0 \leq r_g < r < \infty$, $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$, $-\infty < t < \infty$, r_g representing the gravitational radius of the point source. According to GRT, the value of this gravitational radius depends on the relation between q^2 and m^2 in the following manner:

$$r_g = \begin{cases} m+\sqrt{m^2-q^2}, & \text{for } q^2 \leq m^2 \\ 0 & \text{, for } q^2 > m^2 \end{cases} \tag{4.2}$$

We observe that the function r_g depending on q^2 and m^2 has a discontinuity in $q^2 = m^2$.

The static and spherically symmetric electrogravitational field (4.1) has been obtained solving the coupled system of Einstein's Eqs. and Maxwell's Eqs. (3.7), (3.8). In the selected system of coordinates the magnetic field B^α is zero, an only the radial component $E_1 = E(r)$ of the electric field is non-vanishing, depending only on the coordinate r. From (3.3) we can conclude that the only nonzero components of the electromagnetic tensor and of its dual are:

$$F^{23}(r) = -F^{32}(r) = \hat{F}_{14}(r) = -\hat{F}_{41}(r) = E(r). \tag{4.3}$$

The expression of the unknown function $E(r)$ has been obtained solving (3.7), (3.8), with $J^n \equiv 0$. This $E(r)$ has been introduced in the expression (3.10), thus getting the energy-momentum tensor of the considered electromagnetic field. Finally, solving Einstein's Eqs. the solution (4.1) has been obtained.

Let us now consider the problem of finding the electrogravitational field produced by a charged mass point according to RTG. We must solve the system of Eqs. (2.3), (2.4), (3.7), (3.8) in terms of the coordinate of the underlying Minkowski space-time. Only those solutions that satisfy CP can represent the physical acceptable fields.

I suppose that the spherical coordinates $\{r, \theta, \varphi, t\}$ centered in the charged mass point are the coordinates in the underlying Minkowski space-time. The solution (4.1) satisfies Eqs. (2.3) (without the mass terms), (2.4), (3.7), (3.8). I verify now if this solution satisfies (2.4) .

The metric of the Minkowski space-time in which we happen to be when the gravitational field is switched off is:

$$d\sigma^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2. \tag{4.4}$$

From (4.1), the nonzero components of the tensor g^{mn} are:

$$g^{11} = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right),\ g^{22} = -\frac{1}{r^2},\ g^{33} = -\frac{1}{r^2\sin^2\theta},\ g^{44} = \frac{1}{1 - \frac{2m}{r} + \frac{q^2}{r^2}}, \tag{4.5}$$

and the determinant of the metric tensor g_{mn} is given as follows:

$$g = -r^4 \sin^2\theta. \tag{4.6}$$

From (4.5), (4.6) and (2.2) we obtain the nonzero components of $\tilde{g}^{mn}$:

$$\tilde{g}^{11} = -\sin\theta\left(r^2 - 2mr + q^2\right),\ \tilde{g}^{22} = -\sin\theta,\ \tilde{g}^{33} = -\frac{1}{\sin\theta},\ \tilde{g}^{44} = \frac{r^2\sin\theta}{1 - \frac{2m}{r} + \frac{q^2}{r^2}}. \tag{4.7}$$

Taking into account (4.4), the coefficients of the metric connection generated by γ_{mn} are:

$$\gamma^1_{22} = -r,\ \gamma^1_{33} = -r\sin^2\theta,\ \gamma^2_{12} = \gamma^3_{13} = \frac{1}{r},\ \gamma^2_{33} = -\sin\theta\cos\theta,\ \gamma^3_{23} = \cot\theta. \tag{4.8}$$

According to (4.7), (4.8), making $n = 1$ in the system (2.4'), we obtain an equation which is not fulfilled.

So, the solution (4.1) written in the spherical coordinates $\{r, \theta, \varphi, t\}$ with the metric tensor of the Minkowski space-time having the form (4.4), does not satisfy Eq. (2.4); hence it is not an admissible solution in RTG.

For finding an admissible solution in RTG, I use the same procedure as Logunov & Mestvirishvili (1989), Chapter 13. Thus, I am looking for a system of coordinates $\{\eta^i\} = \{R, \Theta, \Phi, T\}$ in which Eqs. (2.3) (without mass terms), (3.7), (3.8) are fulfilled, Eqs. (2.4) establishing a one to one relationship with a nonzero Jacobian, between the sets of coordinates $\{\eta^i\}$ and $\{\xi^i\}=\{r, \theta, \varphi, t\}$, in the Minkowski space-time. I shift from the variables $\{\xi^i\}$ to the variables $\{\eta^i\}$ assuming that:

$$R = R(r), \Theta = \theta, \Phi = \varphi, T = t. \tag{4.9}$$

The function $R(r)$ must satisfy the following restrictions:

$$R(r) > 0, \text{ for } r > r_g \quad , \quad \frac{dR}{dr} > 0, \text{ for } r > r_g \tag{4.10}$$

The transformation is made in such a way that when the gravitational field disappears the metric of the underlying Minkowski space-time takes the form:

$$d\sigma^2 = dt^2 - dR^2 - R^2 d\theta^2 - R^2 \sin^2\theta d\varphi^2. \tag{4.11}$$

The system of Eqs. (2.4) makes possible the finding of the explicit form of the function $R(r)$. Eqs. (2.4) can be written in the following form (see the relations (13.17), (13.22), from Logunov & Mestvirishvili 1989):

$$\frac{1}{\sqrt{-g(\xi)}}\frac{\partial}{\partial \xi^m}\left(\sqrt{-g(\xi)}g^{mn}(\xi)\frac{\partial \eta^p}{\partial \xi^n}\right) = -\gamma^p_{mn}(\eta)\frac{\partial \eta^m}{\partial \xi^i}.\frac{\partial \eta^n}{\partial \xi^j}g^{ij}(\xi). \tag{4.12}$$

From (4.11) the nonzero components $\gamma^p_{mn}(\eta)$ have the expressions:

$$\gamma^1_{22} = -R,\ \gamma^1_{33} = -R\sin^2\theta,\ \gamma^2_{12} = \gamma^3_{13} = \frac{1}{R},\ \gamma^2_{33} = -\sin\theta\cos\theta,\ \gamma^3_{23} = \cot\theta. \tag{4.13}$$

For $n = 2,3,4$ taking into account (4.5), (4.6), (4.13), Eqs. of the system (2.4') become identities, while for $n = 1$ Eq. assumes the form:

$$\frac{d^2R(r)}{dr^2}(r^2 - 2mr + q^2) + 2(r-m)\frac{dR(r)}{dr} - 2R(r) = 0, \text{ for } r > r_g. \tag{4.14}$$

Here the gravitational radius r_g is given by the expressions (4.2).

The solution of this Eq. depends essentially on the relation between q^2 and m^2 (see Soós and me 2000, Section 4).

5 Case $q^2 \leq m^2$

The solution of Eq. (4.14) obtained in this case is the following (see Soós and me 2000, Section 4):

$$R(r) = r - m, \text{ for } r > r_g = m + \sqrt{m^2 - q^2} \tag{5.1}$$

the admissible minimum value for $R(r)$ being:

$$R_g = \sqrt{m^2 - q^2}. \tag{5.2}$$

R_g represents the gravitational radius of our point source in the system of coordinates $\{\eta^i\}$.

From the tensor transformation law, the nonzero components of the effective Riemannian metric g_{mn} in the new coordinate system $\{\eta^i\}$=$\{R, \theta, \varphi, t\}$ are:

$$g_{11} = -\frac{(R+m)^2}{R^2 - m^2 + q^2},\ g_{22} = -(R+m)^2,\ g_{33} = -(R+m)^2\sin^2\theta,\ g_{44} = \frac{R^2 - m^2 + q^2}{(R+m)^2}. \tag{5.3}$$

The above solution was obtained for the first time by Karabut & Chugreev (1987); E.Soós and me (2000) have obtained the same result.

The analysis of the problem in the framework of RTG doesn't stop here. A necessary condition for that the solution (5.3) has physical sense, is CP. Allowing for the expression (4.11) of the underling Minkowski space-time, I choose the Minkowski isotropic vector $u = (0, 1, 0, R(r))$. The condition (2.6) becomes:

$$(R^2 - m^2 + q^2)R^2 \leq (R+m)^4, \quad \text{for} \quad R > R_g, \tag{5.4}$$

R_g being given by (5.2). It is easy to verify that this restriction is fulfilled.

Also, for the Minkowski isotropic vector $u = (1, 0, 0, 1)$, the causality condition:

$$(R^2 - m^2 + q^2)^2 \leq (R + m)^4, \quad \text{for} \quad R > R_g, \tag{5.5}$$

must be satisfied. Because (5.4) is valid, the inequality (5.5) is obviously satisfied.

So, the solution (5.3) represents the physically acceptable solution in RTG.

6 Case $q^2 > m^2$

According to the empirical values of q and m, for a single electron just this case is true. So, it's important to see how looks the solution in this case. This case was firstly analysed in RTG, by Soós and by me in (2000).

Integrating Eq. (4.14) and choosing one of the two constants of integration which appear, in such way that for r tends to ∞ , $\frac{R(r)}{r}$ tends to 1, we get:

$$R(r) = r - m + C\left(1 - \frac{r-m}{p}\arctan\frac{p}{r-m}\right), \text{ for } r > r_g, \tag{6.1}$$

where $p = \sqrt{q^2 - m^2}$ and C is a real constant.

The components of the effective Riemannian space-time , in the system of coordinates $\{\eta^i\}$ are the following:

$$g_{11} = \left(\frac{dr(R)}{dR}\right)^2\left(-\frac{1}{1 - \frac{2m}{r(R)} + \frac{q^2}{r^2(R)}}\right),\ g_{22} = -r^2(R),\ g_{33} = -r^2(R)\sin^2\theta,\ g_{44} = 1 - \frac{2m}{r(R)} + \frac{q^2}{r^2(R)} \tag{6.2}$$

the function $r(R)$ being implicitly given by (6.1).

Eqs. (2.4) being general covariant, the system of coordinates $\{\eta^i\}$ is not a privileged one. Thus, the solution (4.1) is also a solution of this system of Eqs., but the system of Minkowski coordinates in which is written this solution, is one in which the Minkowskian metric does not have the form (4.4) but the form:

$$d\sigma^2 = dt^2 - \left(\frac{dR(r)}{dr}\right)^2 dr^2 - R^2(r)d\theta^2 - R^2(r)\sin^2\theta d\varphi^2, \tag{6.3}$$

$R(r)$ being explicitly given by (6.1).

Therefore, the solution obtained in this case can be written either in the form (6.2) in the system of coordinates for which the underlying Minkowskian metric has the form (4.11), or in the form (4.1) in the system of coordinates for which the underlying Minkowskian metric is (6.3). Soós E. and me (2000) have considered the form (6.2), (4.11).

Apparently, our problem has a family of solutions depending on the parameter C. Any way, the obtained solutions must satisfy CP in RTG. Because we don't have the explicit form of the inverse function $r(R)$, I verify if this principle is satisfied, using the form (4.1), (6.3) of the solutions.

Considering the Minkowskian isotropic vector $u = (0, 1, 0, R(r))$,the causality condition (2.6) becomes:

$$\left(r^2 - 2mr + q^2\right)R^2(r) \leq r^4 \quad \text{for } r > r_g,\ R > R_g, \tag{6.4}$$

where $R_g \geq 0$ is the lower admissible bound for the function $R(r)$. This value must be also determined.

Also, for the Minkowskian isotropic vector $u = \left(1, 0, 0, \frac{dR(r)}{dr}\right)$ the following inequality must be valid:

$$\left(r^2 - 2mr + q^2\right)^2 \left(\frac{dR(r)}{dr}\right)^2 \leq r^4 \quad \text{for } r > r_g,\ R > R_g. \tag{6.5}$$

For the considered case $q^2 > m^2$, according to (4.2), $r_g = 0$. I'll show that if we want the causality conditions (6.4), (6.5) be satisfied, r_g can't be zero. So, the solution obtained by Soós and by me (2000) doesn't fulfill CP in RTG. This is the main reason for what I have reanalysed the problem considered by us.

Indeed, if $r_g = 0$ and in (6.4) we make r tends to zero and we get $R(0) = 0$. This yields:

$$C = \frac{m}{1 - \frac{m}{p}\arctan\frac{p}{m}} \equiv C_1. \tag{6.6}$$

And if in (6.5) we make r tends to zero, we find $\frac{dR(0)}{dr} = 0$, implying:

$$C = \frac{1}{-\frac{1}{p}\arctan\frac{p}{m} + \frac{m}{m^2+p^2}} \equiv C_2. \tag{6.7}$$

It's easy to see that $C_1 < C_2$. Hence, we obtain a contradiction, since from (6.6), (6.7) we must have $C_1 = C_2$. It results that the restrictions (6.4), (6.5) could be fulfilled only if :

$$r_g > 0. \tag{6.8}$$

Now, I am looking for the value of this r_g.

I return to the conditions which must be fulfilled by the function $R(r)$. The function $R(r)$ has the analytical expression (6.1), and must satisfy the conditions (4.10), (6.4), (6.5). The real constant C will be determined such that the positive and increasing function $R(r)$ gets in the possible minimum value $r = r_g$ its possible minimum value denoted by R_g.

From the analytical expression (6.1) of $R(r)$, we notice that this function has the straight line $R(r) = r - m$ like oblique asymptote at ∞. Allowing for the fact that the following function is positive:

$$f(r) = 1 - \frac{r-m}{p}\arctan\frac{p}{r-m} \geq 0, \text{ for } r > 0, \tag{6.9}$$

from (6.1) we get :

$$R(r) \leq r - m \qquad \text{if and only if} \qquad C \leq 0 \tag{6.10}$$

and

$$R(r) > r - m \qquad \text{if and only if} \qquad C > 0. \tag{6.11}$$

I'll present separately the two possibilities (6.10), (6.11), respectively.

a) $C \leq 0$

In this case, taking into account the expression (6.1) of the function $R(r)$, the condition $(4.10)_1$ which this function must satisfy and the relation (6.9), we conclude that:

$$r > m. \tag{6.12}$$

Deriving the function (6.1), we get:

$$\frac{dR(r)}{dr} = 1 + C\left(-\frac{1}{p}\arctan\frac{p}{r-m} + \frac{r-m}{(r-m)^2+p^2}\right). \tag{6.13}$$

Since, the following function is negative:

$$h(r) = -\frac{1}{p}\arctan\frac{p}{r-m} + \frac{r-m}{(r-m)^2+p^2} \leq 0, \text{ for } r > 0, \tag{6.14}$$

in the considered case we obtain:

$$\frac{dR(r)}{dr} \geq 1, \text{ for } r > r_g. \tag{6.15}$$

So, the condition $(4.10)_2$ is obviously fulfilled.

The causality condition (6.5) must be also valid for $r > r_g$, $R > R_g$, and allowing for (6.15) we find:

$$r \geq \frac{q^2}{2m}. \tag{6.16}$$

In view of (6.10), (6.16) and of the fact that $r - m < r$, the causality condition (6.4) is satisfied.

The conditions (6.12), (6.16)are necessary conditions. For that $(4.10)_2$ and (6.5) to be valid, the real constant C would have to satisfy:

$$C > \frac{m-r}{1 - \frac{r-m}{p}\arctan\frac{p}{r-m}}, \text{ for } r > r_g, \tag{6.17}$$

and respectively:

$$C \geq \frac{2mr - q^2}{(r^2 - 2mr + q^2)\left(-\frac{1}{p}\arctan\frac{p}{r-m} + \frac{r-m}{(r-m)^2+p^2}\right)}, \text{ for } r > r_g. \tag{6.18}$$

According to (6.12), (6.16), we find:

$$r_g = \begin{cases} m, & \text{for } m^2 < q^2 < 2m^2 \\ \frac{q^2}{2m}, & \text{for } 2m^2 \leq q^2. \end{cases} \tag{6.19}$$

Taking into account the criterion for choosing the real constant C, the inequalities (6.17), (6.18), where r_g has the value (6.19), we finally get:

$$C = 0. \tag{6.20}$$

Substituting (6.20) into (6.1), we find in this case :

$$R(r) = r - m, \text{ for } r > r_g. \tag{6.21}$$

r_g being given by (6.19).

b) $C > 0$

In this case, r also can not take values smaller than m. Indeed, if r would take the values smaller than m, from (6.13), the point $r = m$ would be a return point for $R(r)$, so, $R(r)$ wouldn't be strictly increasing function.

Besides, in this case the condition $(4.10)_2$ is not fulfilled. Indeed, from (6.13), allowing for the fact that the function $h(r)$ from (6.14) is monotonously increasing function, and tends to zero for r tends to ∞, we get that there exist $r > r_g \geq m$ such that $\frac{dR(r)}{dr} = 0$.

So, for $C > 0$, the function $R = R(r)$ can't be strictly increasing function for $r > r_g$.

Summing up, if the relation between $q^2$and m^2 is $q^2 > m^2$, the only function, in the form (6.1), which satisfies the restrictions (4.10), (6.4), (6.5) is the function (6.21), where r_g is given by (6.19).

Then the analytical expression of $R = R(r)$ is the same as in the case $q^2 \leq m^2$, but its domain of definition is different.

7 Conclusions

We can conclude that the solution of our problem according to RTG is unique for all order relations which can exist between m^2 and q^2. The analytical expression of the solution doesn't depend on the relation between m^2 and q^2, but its domain of definition, i.e. r_g, depends on this relation. On the basis of (5.1), (6.21), (6.19), we can write:

$$R(r) = r - m, \text{ for } r > r_g. \tag{7.1}$$

where

$$r_g = \begin{cases} m + \sqrt{m^2 - q^2}, \text{ for } q^2 \leq m^2 \\ m, \text{ for } m^2 < q^2 < 2m^2 \\ \frac{q^2}{2m}, \text{ for } 2m^2 \leq q^2. \end{cases} \tag{7.2}$$

We notice that the function r_g depending on q^2 and m^2 is a continuous one.

The expression of the effective Riemannian metric has in the system of coordinates $\{\xi^i\} = \{r, \theta, \varphi, t\}$ the same form (4.1) like in GRT. But in RTG , these coordinates are the spatial-temporal coordinates in the Minkowski universe. The line element of this universe doesn't have the form (4.4) but the form (6.3). Substituting (7.1) into (6.3), we get this form:

$$d\sigma^2 = dt^2 - dr^2 - (r-m)^2 d\theta^2 - (r-m)^2 \sin^2\theta d\varphi^2. \tag{7.3}$$

Comparing (4.2) and (7.2), I also stress the difference between the gravitational radius in this two theories.

We can also write the components of the effective Riemannian metric in the system of coordinates $\{\eta^i\} = \{R, \theta, \varphi, t\}$,in which the Minkowskian line element has the form (4.11). These are:

$$g_{11} = -\frac{(R+m)^2}{R^2 - m^2 + q^2}, \; g_{22} = -(R+m)^2, \; g_{33} = -(R+m)^2 \sin^2\theta, \; g_{44} = \frac{R^2 - m^2 + q^2}{(R+m)^2}. \tag{7.4}$$

From (7.1), (7.2), the gravitational radius in the system of coordinates $\{\eta^i\}$ has the expression:

$$R_g = \begin{cases} \sqrt{m^2 - q^2}, \text{ for } q^2 \leq m^2 \\ 0, \text{ for } m^2 < q^2 < 2m^2 \\ \frac{q^2 - 2m^2}{2m}, \text{ for } 2m^2 \leq q^2. \end{cases} \tag{7.5}$$

R_g like function depending on q^2 and m^2, is also a continuous function.

The obtained result shows again the important role played by CP in RTG: indeed the analytical expression of the function $R = R(r)$ is much more simple as that obtained by E Soós and by me (2000).

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References

[1] Ionescu, D. & Soós, E. 2000 Electrogravitational Field Produced by a Charged Mass Point in RTG. *Rev. Roum. Math. P. Appl.*, **45** (2), 251-260, 2000.

[2] Karabut, P. V. & Chugreev Iu. V. 1987 Exterior Axial-Symmetric Solution for a Spinning Charged Body in RTG. *Institute of High-Energy Physics Preprint*, 87-142. Serpuhov.

[3] Logunov, A. A. & Mestvirishvili, M. 1989 *The Relativistic Theory of Gravitation*. Moscow: Mir.

[4] Logunov, A. A. 1997 *Relativistic Theory of Gravity and Mach Principle*. Dubna.

[5] Moller, C. 1972 *The Theory of Relativity*. Oxford: Clarendon Press.

[6] Soós, E. 1992 *Géométrie et électromagnetisme*. University of Timisoara, Math. Dept..

[7] Truesdell, C. & Toupin, R. A. 1960 *The Classical Field Theories*. Handbuch der Physik, **III/1**, pp. 226-858. Berlin, Gottingen, Heidelberg: Springer.

[8] Wang, C. C. 1979 *Mathematical Principles of Mechanics and Electromagnetism, Part B: Electromagnetism and Gravitation*. New York, London: Plenum Press.